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BBG-003-1016002 Seat No. ________

B. Sc. (Sem. IV) (CBCS) Examination

July - 2021

Mathematics : Paper - M - 09 (A)

(Mathematical Analysis - 2 & Abstract Algebra - 2)

Faculty Code : 003
Subject Code : 1016002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instruction: Attempt any five questions.

1. (A) Answer the following questions in briefly 4
   (1) Define . Separated set
   (2) Define : Connected set
   (3) Determine whether the subset {1,2,3} of metric space R is compact or not
   (4) Define compact metric space
   (B) Show that subset R-{1,5} is not connected 2
   (C) State and prove Bolzano-Weirstrass theorem 3
   (D) State and prove theorem of nested intervals 5

2. (A) Answer the following questions in briefly 4
   (1) Define: Totally bounded set
   (2) Define: Disconnected set
   (3) Define: Countable set
   (4) Define : Sequential compact metric space
   (B) Show that every finite subset of a metric space is compact 2
   (C) If F is a closed subset of metric pace X and K is a compact subset of X
   Then prove that F∩K is also compact 3
   (D) Prove that continuous image of connected set is connected 5

3. (A) Answer the following questions in briefly 4
   (1) Define Laplace Transform
   (2) Find $L^{-1}\left(\frac{1}{s-2}\right)$
   (3) Find $L(t^{-1/2})$
   (4) Show that $L(1) = \frac{1}{s}$,where $s > 0$
   (B) Find $L^{-1}\left(\frac{s+2}{(s-2)^3}\right)$ 2
   (C) Find Laplace transform of $\sqrt{t}e^{2t}$ 3
   (D) If $f(t) = e^t$, $t \le 2$
   $= 3$, $t > 2$ then find $L\{f(t)\}$ 5

4. (A) Answer the following questions in briefly 4

(1) Find $L(3^t)$

(2) Find $L^{-1}\left(\frac{1}{s^3}\right)$

(3) Find $L^{-1}\left(\frac{1}{s^2+4}\right)$

(4) Show that $L(t) = \frac{1}{s}$

(B) Find L(2t + 5sin3t) 2

(C) If $L\{f(t)\} = \bar{f}(s)$ then prove that $L\{e^{at} f(t)\} = \bar{f}(s - a)$ 3

(D) Prove that $L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right) = \frac{1}{2a} tsinat$ 5

5. (A) Answer the following questions in briefly 4

(1) Find $L(te^t)$

(2) Write convolution theorem

(3) Find L(tsin 2t)

(4) Find $L\left(\frac{\sin t}{t}\right)$

(B) If $L\{f(t)\} = \bar{f}(s)$ then prove $L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}[\bar{f}(s)]$ 2

(C) Prove that $L^{-1}\left(\log\left(\frac{s+b}{s+a}\right)\right) = \frac{e^{-at} - e^{-bt}}{t}$ 3

(D) Prove that $L^{-1}\left\{\frac{s^2-a^2}{(s^2+a^2)^2}\right\} = t\cos at$ 5

6. (A) Answer the following questions in briefly 4

(1) Find $L(t^2e^{at})$

(2) Find L(tsinht)

(3) Find L(tsin at)

(4) Find $L(t^3e^{-3t})$

(B) If $L\{f(t)\} = \bar{f}(s)$ then prove $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \bar{f}(s)\, ds$ 2

(C) Prove that $L\left\{\frac{e^{-at} - e^{-bt}}{t}\right\} = \log\left(\frac{s+b}{s+a}\right)$ 3

(D) Using convolution theorem, prove $L^{-1}\left\{\frac{1}{(s-1)(s^2+1)}\right\} = \frac{1}{2}(e^t - \sin t - \cos t)$ 5

7. (A) Answer the following questions in briefly 4

(1) Define Principle ideal

(2) Define homomorphism

(3) Define Ring

(4) Define ideal

(B) If $\emptyset: (G,*) \to (G', \Delta)$ is a Homomorphism. Then $\emptyset$(e) = e' where e & e' are identity elements of G & G' respectively. 2

(C) Prove that A Homomorphism $\emptyset: (G,*) \to (G', \Delta)$ is one-one iff $k_{\emptyset}$ ={e} 3

(D) If $\emptyset: (G,*) \to (G',\Delta)$ is a Homomorphism. Then prove that Kernel $K_{\emptyset}$ is a normal Subgroup of G 5

8. (A) Answer the following questions in briefly 4
   (1) Define Epimorphism
   (2) Define Division ring
   (3) Define Field
   (4) Define Kernel of homomorphism

(B) Let $\emptyset: (G,*) \to (G',\Delta)$ is Homomorphism. If $H' \leq G'$ then prove $\emptyset^{-1}(H) \leq G$ 2

(C) Find all homomorphism's of (Z,+) onto (Z,+). 3

(D) State and prove first fundamental theorem of homomorphism 5

9. (A) Answer the following questions in briefly 4
   (1) Define Polynomial
   (2) If polynomial f = (5,0,0,0,0,.. ..) then find order of f
   (3) Define irreducible polynomial
   (4) Define Monic polynomial

(B) Find inverse of quaternion 1 + i + j + k 2

(C) State and prove Remainder theorem of polynomials 3

(D) State and prove division algorithm for polynomials 5

10. (A) Answer the following questions in briefly 4
   (1) Define degree of a polynomial
   (2) Define order of a polynomial
   (3) Define factor polynomial
   (4) Define constant polynomial

(B) If f(x)=(2,3,4,2,0,0....) and g(x)=(4,2,0,0,3,0....)∈ R[x] then find f(x)+g(x). 2

(C) In R[x], $f(x) = 4x^4 - 3x^2 + 1$ is divided by $g(x) = x^3 - 2x + 1$ then find quotient q(x) and remainder r(x) 3

(D) State and prove factor theorem of polynomials 5
